OCR

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4728/01**

MATHEMATICS

Mechanics 1

MONDAY 21 MAY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

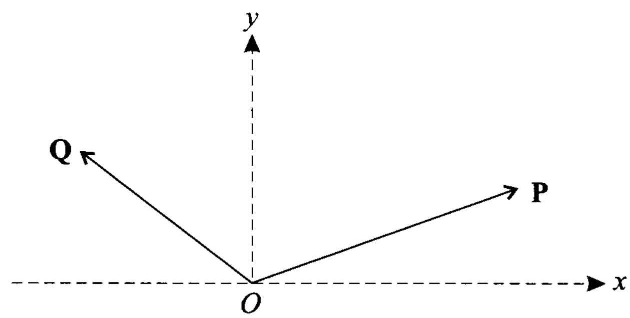
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **6** printed pages and **2** blank pages.

1

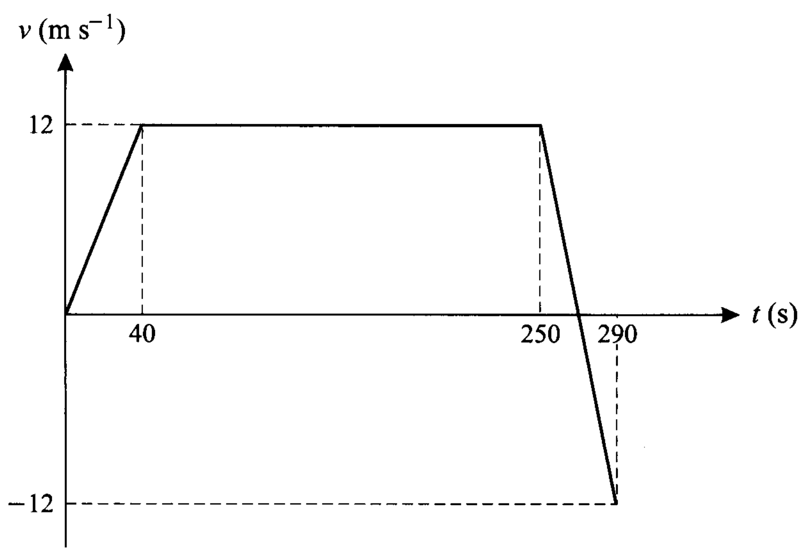


Two horizontal forces **P** and **Q** act at the origin O of rectangular coordinates Oxy (see diagram). The components of **P** in the x- and y-directions are 14 N and 5 N respectively. The components of **Q** in the x- and y-directions are −9 N and 7 N respectively.

(i) Write down the components, in the x- and y-directions, of the resultant of **P** and **Q**. [2]

(ii) Hence find the magnitude of this resultant, and the angle the resultant makes with the positive x-axis. [4]

2



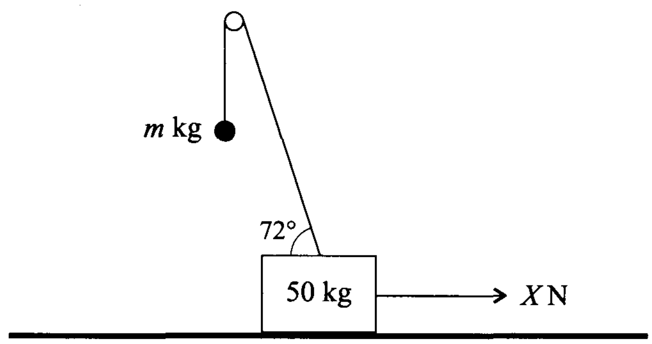
A particle starts from the point A and travels in a straight line. The diagram shows the (t, v) graph, consisting of three straight line segments, for the motion of the particle during the interval $0 \leqslant t \leqslant 290$.

(i) Find the value of t for which the distance of the particle from A is greatest. [2]

(ii) Find the displacement of the particle from A when $t = 290$. [3]

(iii) Find the total distance travelled by the particle during the interval $0 \leqslant t \leqslant 290$. [2]

3



A block of mass 50 kg is in equilibrium on smooth horizontal ground with one end of a light wire attached to its upper surface. The other end of the wire is attached to an object of mass m kg. The wire passes over a small smooth pulley, and the object hangs vertically below the pulley. The part of the wire between the block and the pulley makes an angle of 72° with the horizontal. A horizontal force of magnitude X N acts on the block in the vertical plane containing the wire (see diagram).

The tension in the wire is T N and the contact force exerted by the ground on the block is R N.

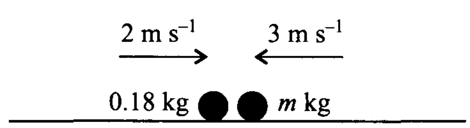
(i) By resolving forces on the block vertically, find a relationship between T and R. [2]

It is given that the block is on the point of lifting off the ground.

(ii) Show that $T = 515$, correct to 3 significant figures, and hence find the value of m. [4]

(iii) By resolving forces on the block horizontally, write down a relationship between T and X, and hence find the value of X. [2]

4



Two particles of masses 0.18 kg and m kg move on a smooth horizontal plane. They are moving towards each other in the same straight line when they collide. Immediately before the impact the speeds of the particles are 2 m s^{-1} and 3 m s^{-1} respectively (see diagram).

(i) Given that the particles are brought to rest by the impact, find m. [3]

(ii) Given instead that the particles move with equal speeds of 1.5 m s^{-1} after the impact, find

(a) the value of m, assuming that the particles move in opposite directions after the impact, [3]

(b) the two possible values of m, assuming that the particles coalesce. [4]

5 A particle P is projected vertically upwards, from horizontal ground, with speed $8.4\,\text{m}\,\text{s}^{-1}$.

(i) Show that the greatest height above the ground reached by P is 3.6 m. [3]

A particle Q is projected vertically upwards, from a point 2 m above the ground, with speed $u\,\text{m}\,\text{s}^{-1}$. The greatest height **above the ground** reached by Q is also 3.6 m.

(ii) Find the value of u. [2]

It is given that P and Q are projected simultaneously.

(iii) Show that, at the instant when P and Q are at the same height, the particles have the same speed and are moving in opposite directions. [6]

6 A particle starts from rest at the point A and travels in a straight line. The displacement s m of the particle from A at time t s after leaving A is given by

$$s = 0.001t^4 - 0.04t^3 + 0.6t^2, \quad \text{for } 0 \leqslant t \leqslant 10.$$

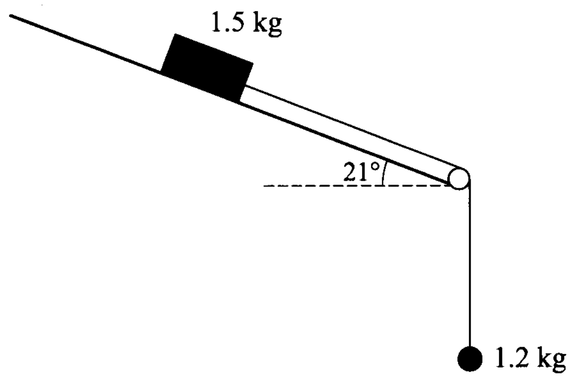
(i) Show that the velocity of the particle is $4\,\text{m}\,\text{s}^{-1}$ when $t = 10$. [3]

The acceleration of the particle for $t \geqslant 10$ is $(0.8 - 0.08t)\,\text{m}\,\text{s}^{-2}$.

(ii) Show that the velocity of the particle is zero when $t = 20$. [5]

(iii) Find the displacement from A of the particle when $t = 20$. [6]

7



One end of a light inextensible string is attached to a block of mass 1.5 kg. The other end of the string is attached to an object of mass 1.2 kg. The block is held at rest in contact with a rough plane inclined at 21° to the horizontal. The string is taut and passes over a small smooth pulley at the bottom edge of the plane. The part of the string above the pulley is parallel to a line of greatest slope of the plane and the object hangs freely below the pulley (see diagram). The block is released and the object moves vertically downwards with acceleration $a\,\text{m}\,\text{s}^{-2}$. The tension in the string is T N. The coefficient of friction between the block and the plane is 0.8.

(i) Show that the frictional force acting on the block has magnitude 10.98 N, correct to 2 decimal places. [3]

(ii) By applying Newton's second law to the block and to the object, find a pair of simultaneous equations in T and a. [5]

(iii) Hence show that $a = 2.24$, correct to 2 decimal places. [2]

(iv) Given that the object is initially 2 m above a horizontal floor and that the block is 2.8 m from the pulley, find the speed of the block at the instant when

(a) the object reaches the floor, [2]

(b) the block reaches the pulley. [4]

1(i)	X = 5 Y = 12	B1 B1 [2]	X=-5 B0. Both may be seen/implied in (ii) No evidence for which value is X or Y available from (ii) award B1 for the pair of values 5 and 12 irrespective of order
(ii)	$R^2 = 5^2 + 12^2$ Magnitude is 13 N $\tan\theta$ = 12/5 Angle is 67.4°	M1 A1 M1 A1 [4]	For using $R^2 = X^2 + Y^2$ Allow 13 from X=-5 For using correct angle in a trig expression **SR:** p=14.9 and Q=11.4 giving R=13+/-0.1 B2, Angle = 67.5+/-0.5 B2

2(i)	250 + ½ (290 – 250) t = 270	M1 A1 [2]	Use of the ratio 12:12 (may be implied), or v = u+at
(ii)	½ x40x12+210x12+½x20x12–½x20x12 or ½ x40x12+210x12 or ½ x(210+250)x12etc Displacement is 2760m	M1 M1 A1 [3]	The idea that area represents displacement Correct structure, ie triangle1 + rectangle2 + triangle3 - |triangle4| with triangle3 = |triangle4|, triangle1 + rectangle2, trapezium1&2, etc
(iii)	appropriate structure, ie triangle + rectangle + triangle + |triangle|, triangle + rectangle + 2triangle, etc Distance is 3000m	M1 A1 [2]	All terms positive Treat candidate doing (ii) in (iii) and (iii) in (ii) as a mis-read.

3(i)	R + Tsin72° = 50g	M1 A1 [2]	An equation with R, T and 50 in linear combination. R + 0.951T = 50g
(ii)	T = 50g/sin72° T = 515 (AG) T = mg m = 52.6	M1 A1 B1 B1 [4]	Using R = 0 (may be implied) and Tsin72° = 50(g) Or better Accept 52.5
(iii)	X = Tcos72° X = 159	B1 B1 [2]	Implied by correct answer Or better

4(i)	*In Q4 right to left may be used as the positive sense throughout.* 0.18 x 2 – 3m = 0 m = 0.12	M1 A1 A1 [3]	For using Momentum 'before' is zero 3 marks possible if g included consistently
(iia)	Momentum after = -0.18 x 1.5 + 1.5m 0.18 x 2 – 3m = -0.18 x 1.5 + 1.5m m = 0.14	B1 M1 A1 [3]	For using conservation of momentum 3 marks possible if g included consistently
(iib)	0.18 x 2 – 3m = (0.18 + m)1.5 m = 0.02 0.18 x 2 – 3m= - (0.18 + m)1.5 m = 0.42	B1ft B1 B1ft B1 [4]	ft wrong momentum 'before' 0 marks if g included

5(i)		M1	Using $v^2 = u^2$ +/– 2gs with v = 0 or u = 0
	$8.4^2 - 2gs_{max} = 0$	A1	
	Height is 3.6m (AG)	A1	
		[3]	
(ii)		M1	Using u^2 =+/- 2g(ans(i) – 2)
	u = 5.6	A1	
		[2]	
(iii)	EITHER (time when at same height)	M1	Using s = ut + ½ at^2 for P *and* for Q, a = +/-g, expressions for s terms must differ
	s+/-2 = 8.4t – ½ gt^2 and		Or 8.4t (– ½ gt^2)=5.6t (– ½ gt^2)+/- 2
	(s+/-2) = 5.6t – ½ gt^2	A1	Correct sign for g, cv(5.6), +/-2 in only one equation
	t = 5/7 (0.714)	A1	cao
		M1	Using v = u +at for P *and* for Q, a = +/-g, cv(t)
	v_P =8.4 -0.714g and v_Q =5.6-0.714g	A1	Correct sign for g, cv(5.6), candidates answer for t (including sign)
	v_P = 1.4 and v_Q = -1.4	A1	cao
		[6]	
	OR (time when at same speed in opposite directions)	M1	Using v = u+at for P *and* for Q, a = +/-g
	v = 8.4 -gt and -v =5.6-gt	A1	Correct sign for g, cv(5.6)
	v = 1.4 {or t = 5/7 (0.714)}	A1	Only one correct answer is needed
	(with v = 1.4)	M1	Using $v^2 = u^2$ + 2as for P *and* for Q, a = +/-g, cv(v)
	$1.4^2 = 8.4^2 - 2gs_P$ and $(-1.4)^2 = 5.6^2 - 2gs_Q$	A1	Correct sign for g, cv(5.6), candidate's answer for v (including - for Q)
	s_P = 3.5 and s_Q = 1.5	A1	cao
	{(with t=5/7)	M1	Using s = ut + ½ at^2 for P *and* for Q, a = +/-g, cv(t)
	s = 8.4x0.714 – ½ gx0.714^2 and s = 5.6x0.714 – ½ gx0.714^2	A1	Correct sign for g, cv(5.6), candidate's answer for t (including sign of t if negative)
	s_P = 3.5 and s_Q = 1.5	A1	cao}
	OR (motion related to greatest height and verification)	M1	Using v = u+at t for P *and* for Q, a = +/-g
	0 = 8.4 -gt and 0 =5.6-gt		
	t = 6/7 and t = 4/7	A1	Both values correct
	v_P =8.4 -0.714g and v_Q =5.6-0.714g		mid-interval t (6/7+4/7)/2 = 0.714
	{0 = v_P - g/7 and v_Q = 0 +g/7}		{Or semi-interval = 6/7-4/7)/2=1/7}
	v_P = 1.4 and v_Q = -1.4	A1	cao
	s_P = 8.4x0.714 – ½ gx0.714^2 and s_Q = 5.6x0.714 – ½ gx0.714^2	M1	s = ut + ½ at^2 for P *and* for Q, correct sign for g, cv(5.6) and cv(t)
	{ s_P = 0/7 – ½(- g)x$(1/7)^2$ and s_Q = 0/7 + ½ gx$(1/7)^2$}	A1	{s = vt - ½ at^2 for P *and* s = ut + ½ at^2 for Q}
	s_P = 3.5 s_Q = 1.5		
	{ s_P = 0.1 s_Q = 0.1}	A1	cao
			continued

5(iii)	OR (without finding exactly where or when)	M1	Using $v^2 = u^2 + 2as$ for P *and* for Q, a = +/-g, cv(5.6), different expressions for s.
co*nt*	$v_P^2 = 8.4^2 - 2g(s+/-2)$ and		Correct sign for g, cv(5.6), (s+/-2) used only once
	$v_Q^2 = 5.6^2 - 2g[(s+/-2)]$	A1	cao. Verbal explanation essential
	$v_P^2 = v_Q^2$ for all values of s so that the speeds are always the same at the same heights.	A1	Using v = u+at t for P *and* for Q, a = +/-g
		M1	Correct sign for g, correct choice for velocity of zero, cv(5.6)
	0 = 8.4 -gt and 0 =5.6-gt	A1	
	$t_P = 6/7$ and $t_Q = 4/7$ means there is a time interval when Q has started to descend but P is still rising, and there will be a position where they have the same height but are moving in opposite directions.	A1	cao. Verbal explanation essential

6(i)		M1	For differentiating s
	$v = 0.004t^3 - 0.12t^2 + 1.2t$	A1	Condone the inclusion of +c
	$v(10) = 4 - 12 + 12 = 4\text{ms}^{-1}$ (AG)	A1	Correct formula for v (no +c) and t=10
		[3]	stated sufficient
(ii)		M1	For integrating a
	$v = 0.8t - 0.04t^2$ (+ C)	A1	
	8 - 4 + C = 4	M1*	Only for using v(10) = 4 to find C
	$v = 0.8 \times 20 - 0.04 \times 20^2$ (+ C)	M1	
	v(20) = 16 – 16 = 0 (AG)	DA1	Dependant on M1*
		[5]	
(iii)		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$ (+K)	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For using S(10) = 30 to find K
	40 – 40/3 + K = 30 ➔ K = 10/3	A1	Not if S includes ct term
	S(20) = 160 – 320/3 + 10/3 = 56.7m	B1	Accept 56.6 to 56.7, Adding 30 subsequently is not isw,
	OR	[6]	hence B0
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
		M1	Using limits of 10 and 20 (limits 0, 10 M0A0B0)
	S(20) - S(10) = 26.6, 26.7	A1	For 53.3 - 26.7 or better (Note S(10) = 26.7 is fortuitously correct M0A0B0)
	displacement is 56.7m	B1	Accept 56.6 to 56.7

7(i)	R = 1.5gcos21°	B1	
		M1	For using F = μR
	Frictional force is 10.98N (AG)	A1 [3]	Note 1.2gcos21=10.98 fortuitously, B0M0A0
(ii)		M1	For obtaining an N2L equation relating to the block in which F, T, m and a are in linear combination or For obtaining an N2L equation relating to the object in which T, m and a are in linear combination
	T + 1.5gsin21° – 10.98 = 1.5a	A2	-A1 for each error to zero
	1.2g – T = 1.2a	A2	-A1 for each error to zero
		[5]	Error is a wrong/omitted term, failure to substitute a numerical value for a letter (excluding g), excess terms. Minimise error count.
(iii)	T – 1.5a = 5.71 and 1.2a + T = 11.76	M1	For solving the simultaneous equations in T and a for a.
	a = 2.24 (AG)	A1 [2]	Evidence of solving needed
(iva)	v^2 = 2 x 2.24 x 2	M1	For using v^2 = 2as with cv (a) or 2.24
	Speed of the block is 2.99ms^{-1}	A1 [2]	Accept 3
(ivb)		M1	For using T = 0 to find a
	a = -3.81	A1	
	$v^2 = 2.99^2$ + 2 x (-3.81) x 0.8	M1	For using $v^2 = u^2$ + 2as with cv(2.99) and s = 2.8 - 2 and any value for a
	Speed of the block is 1.69ms^{-1}	A1 [4]	Accept art 1.7 from correct work